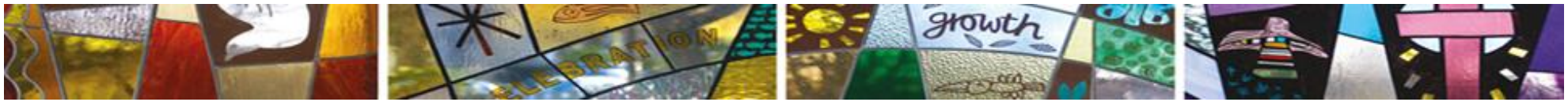


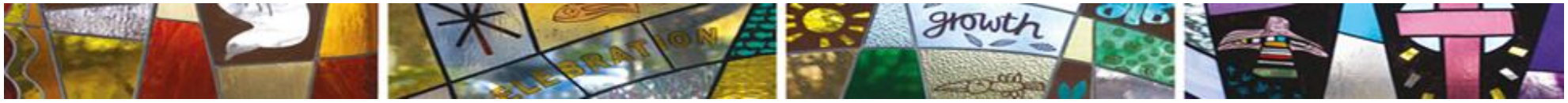
St. Peter's
Church of England Primary School

LOVE LEARN SHINE

Shine in the light and love of God

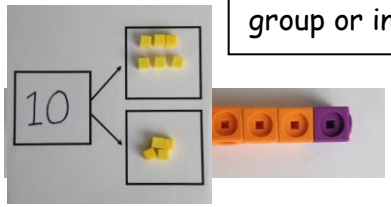
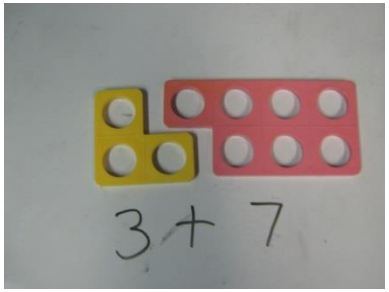
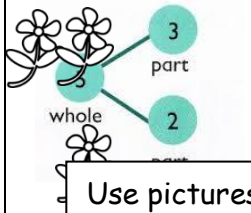
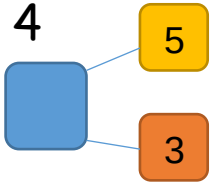
Curriculum Progressions

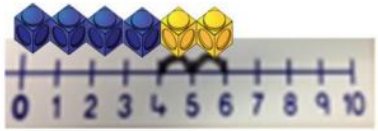
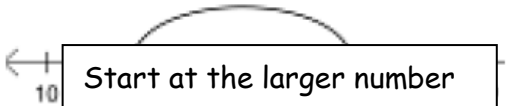
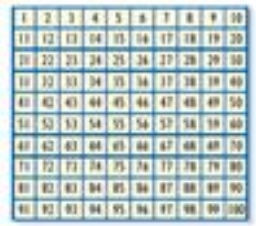
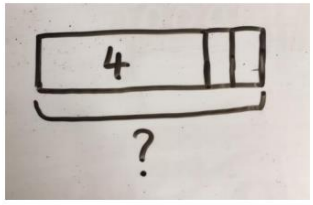
Progression in Mathematical Calculation

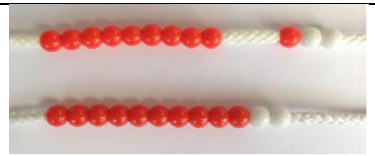
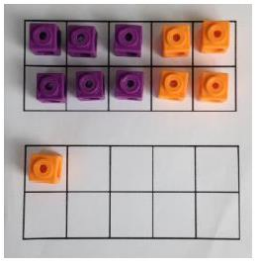
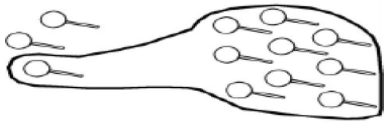
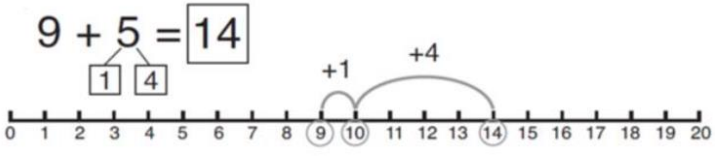
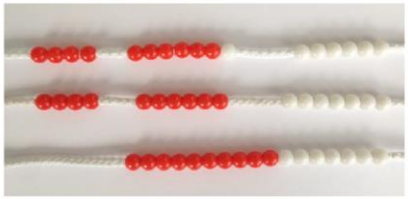
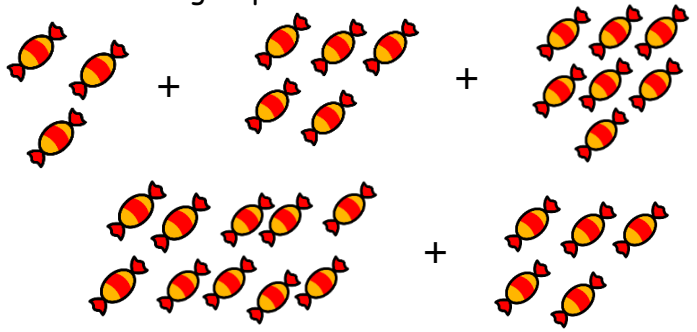
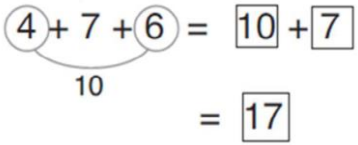


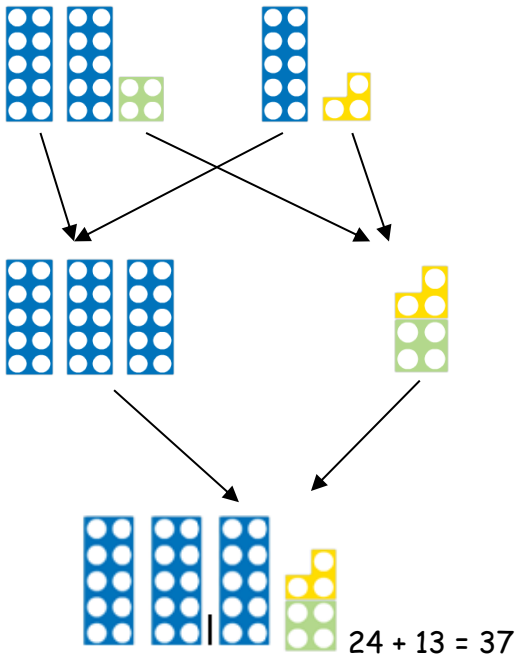
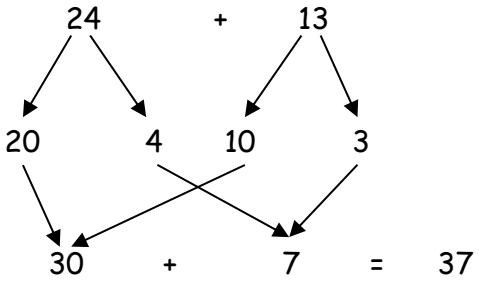
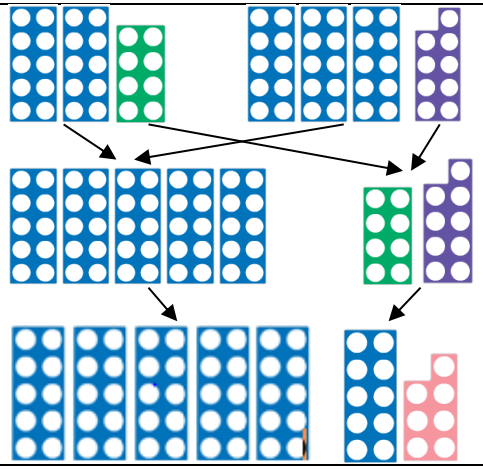
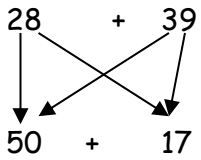
Addition

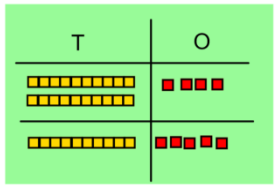
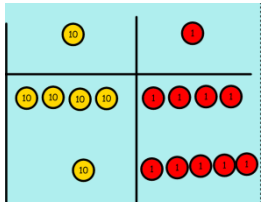
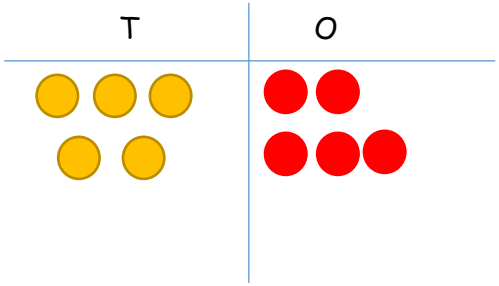
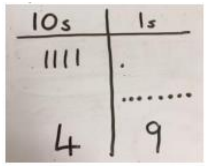
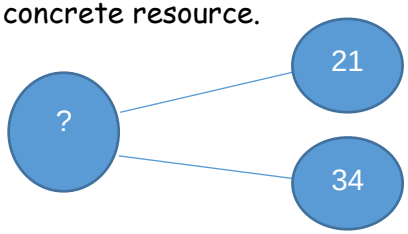
Key Vocabulary: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to', 'is the same as', addend

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model (aggregation)	  Use cubes to add two numbers together as a group or in a bar.  	 Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 Addend + Addend = Sum 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.

	Use other resources too e.g. eggs, shells, teddy bears, cars.		
Starting at the larger number and counting on (augmentation)	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Use cubes and Numicon too. 	12 + 5 = 17  Start at the larger number on the number line or hundred square and count on in ones or in one jump to find the answer.  A bar model that encourages the children to count on, rather than count all. 	5 + 12 = 17 *'Place the largest number in your head and count on the smaller number to find your answer.'* What is 5 more than 12? What is the sum of 12 and 5? What is the total of 5 and 12?

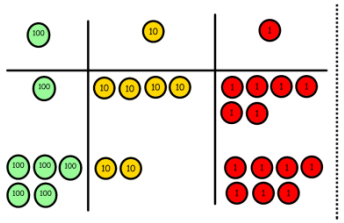
Regrouping to make 10.	 6 + 5 = 11  Start with the larger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10.  Children to draw counters/cubes on a tens frame.	7 + 4 = 11 *'If I am at seven, how many more do I need to make 10? How many more do I add on now?'* Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4
Adding three single digits N.B. Making 10 should be the primary strategy.	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7. 	Add together three groups of objects. Draw a picture to recombine the groups to make 10. 	 Combine the two numbers that make 10 and then add on the remainder.

Partitioning and recombining		24 + 13 =  Use jotting alongside apparatus e.g. Numicon, base 10	24 = 20 + 4 13 = 10 + 3 30 + 7 = 37 Ongoing dialogue which is not necessary to record.
		<u>Partitioning</u> 28 + 39  50 + 17 = 67	28 = 20 + 8 39 = 30 + 9 50 + 17 = 67 Ongoing dialogue which is not necessary to record.

		Jottings alongside use of apparatus.	
Column method - no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters or Base 10 e.g. lines of tens and dots or crosses for ones.   21 + 34 = 55 Prove it using a mathematical diagram or concrete resource.  	Calculations: 21 + 34 = 2 1 + 3 4  = 21 + 34 Calculate the sum of twenty-one and thirty-four. Word Problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children are there in total?

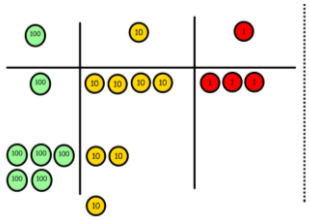
Column method - regrouping

Make both numbers on a place value grid.



146
+ 527

Add up the ones and exchange 10 ones for one 10.



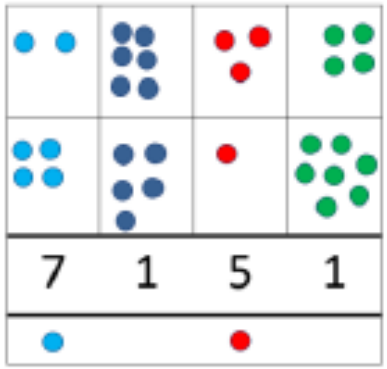
146
+ 527

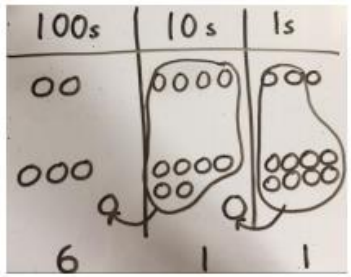
Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.





A place value chart can be used to draw the counters, circling when they make an exchange.

Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

20 + 5
40 + 8
60 + 13 = 73

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

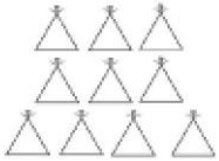
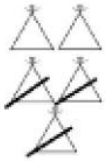
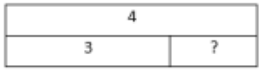
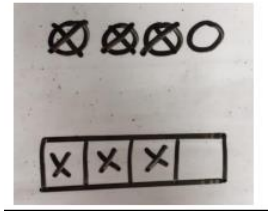
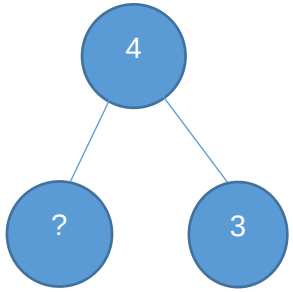
536
+ 85
621
1 1

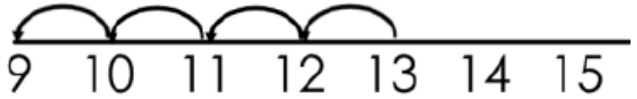
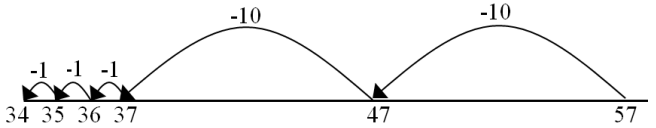
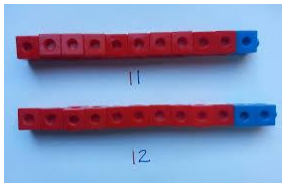
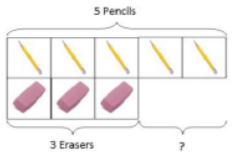
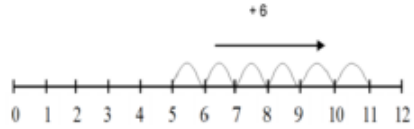
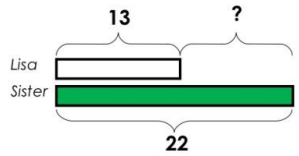
72.8
+ 54.6
127.4
1 1

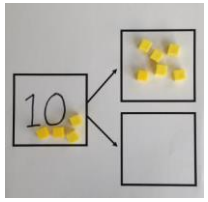
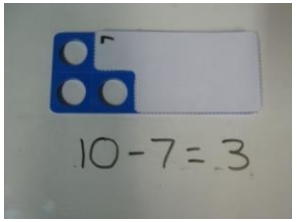
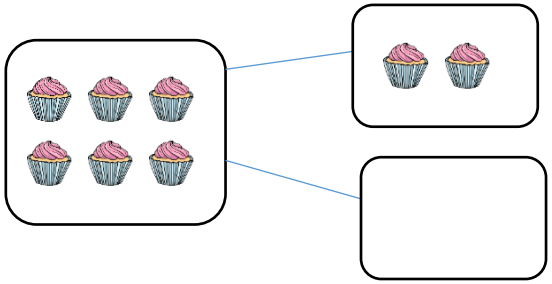
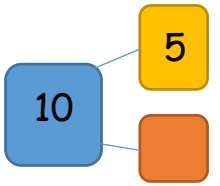
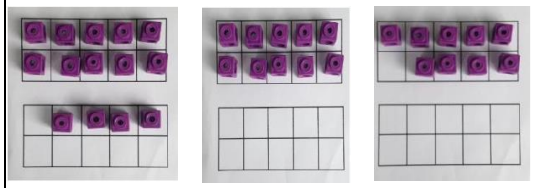
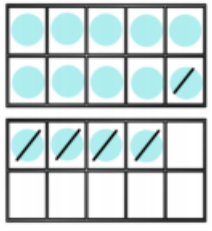
	£	2	3	.	5	9
+	£		7	.	5	5
	£	3	1	.	1	4
		1	1		1	

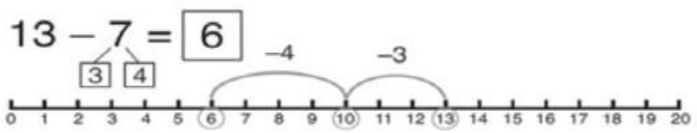
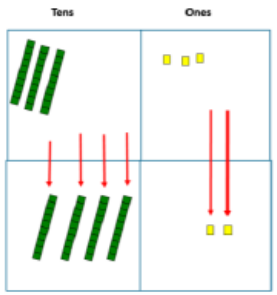
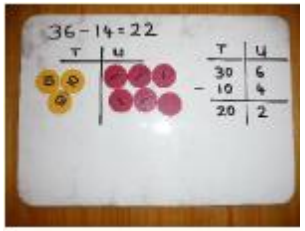
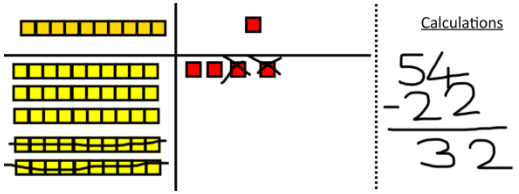
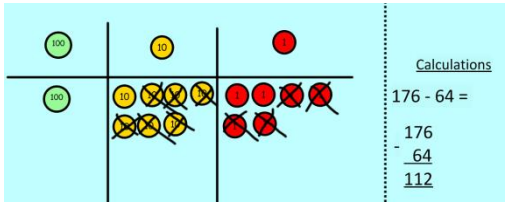
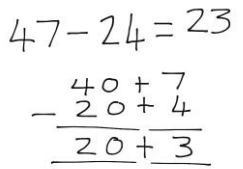
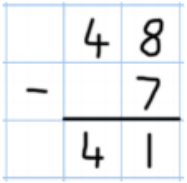
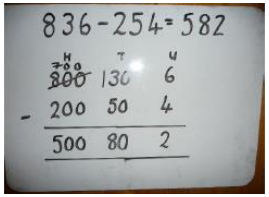
Subtraction

Key Vocabulary: take away, less than, the difference, subtract, minus, fewer, decrease, subtrahend, minuend, wholes and parts

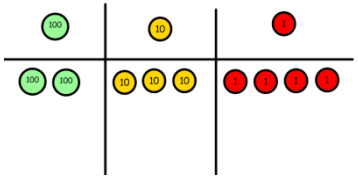
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones Physically taking away and removing objects from a whole	Use physical objects e.g. ten frames, Numicon, cubes and other items such as beanbags could be used.  6 – 2 = 4   Subtraction as 'chopping off'	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.   15 – 3 = 12    	18 – 3 = Minuend – subtrahend = Difference  = 18 - 3

Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Children to represent the calculation on a number line or number track and show their jumps. A hundred square can also be used.  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Encourage the use of an empty number line.
Finding the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference.  Use basic bar models with items to find the difference.	 Count on to find the difference. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  Draw bars to find the difference between 2 numbers. Cuisenaire rods are excellent.	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 – 6 = 8 – 5 = 7 – 4 = have the same difference. Word Problems:

			Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches they have.
Part-Part Whole Model	 Link to addition - use the part whole model to help explain the inverse between addition and subtraction. *If 10 is the whole and 7 is one of the parts. What is the other part?* 10 - 7 = 	Use a pictorial representation of objects to show the part-part whole model. 	 Move to using numbers within the part whole model.
Making 10	14 – 9 =  *Make 14 on the tens frame. Take away the four first to make 10 and then takeaway one more so you have*	Children to represent the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend. 14 – 5 = 9 4 1 14 – 4 = 10

	taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	10 – 1 = 9 *How many do we take off to reach the next 10?* *How many do we have left to take off?*
Column method - without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	  Draw the Base 10 or place value counters alongside the written calculation to help to show working. The Base 10 can also be represented pictorially using lines for tens and dots or crosses for ones.	 This will lead to a clear written column subtraction. 
Column method - with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.	Represent the Base 10 pictorially, remembering to show the exchange. 	

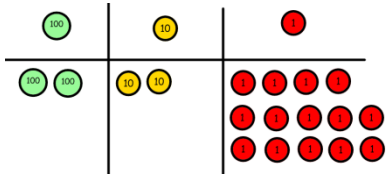
Make the larger number with the place value counters.



Calculations

234
- 88

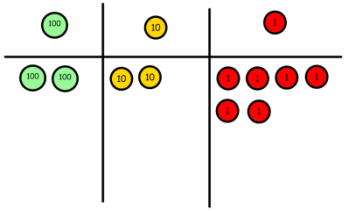
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

234
- 88

Now I can subtract my ones.

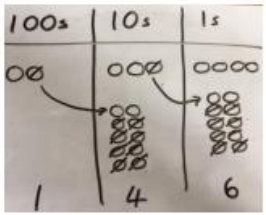


Calculations

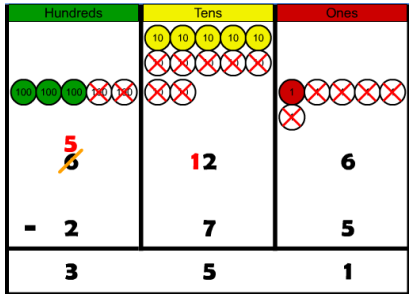
234
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

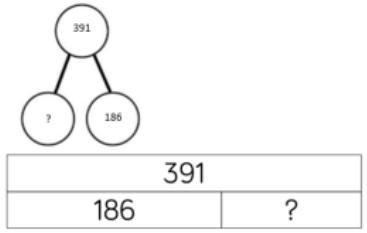


When confident, children can find their own way to record the exchange/regrouping.

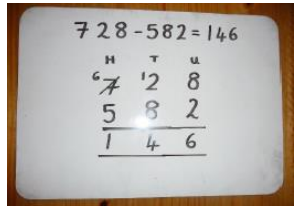


Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

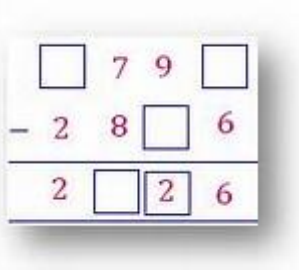


Children can start their formal written method by partitioning the number into clear place value columns.

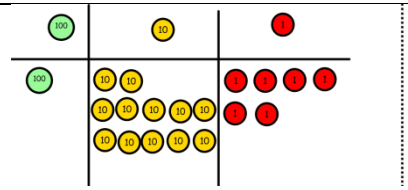
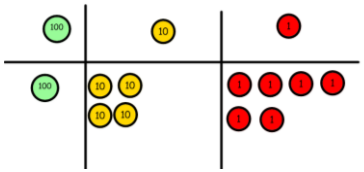


Children must understand what has happened when they have crossed out digits.

Missing Digit Calculations:

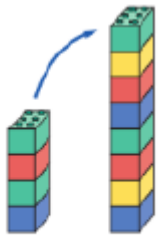
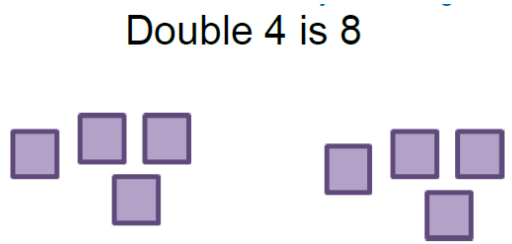
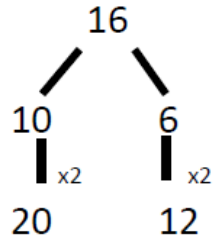
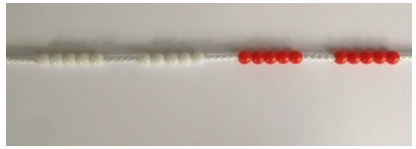
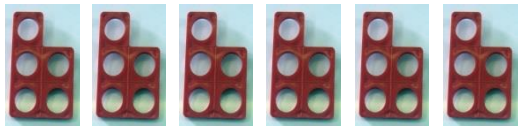
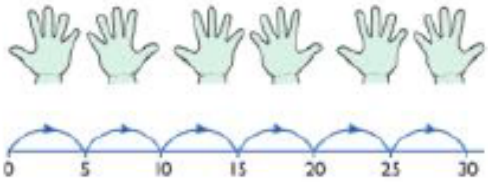
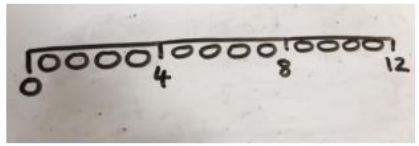


Word Problems:
Raj spent £391, Timmy spend £186. How much more did Raj spend?

	 Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations ~~234~~ - 88 146		

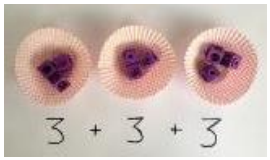
Multiplication

Key Vocabulary: double, times, multiplied by, the product of, groups of, lots of, equal groups, factor, product

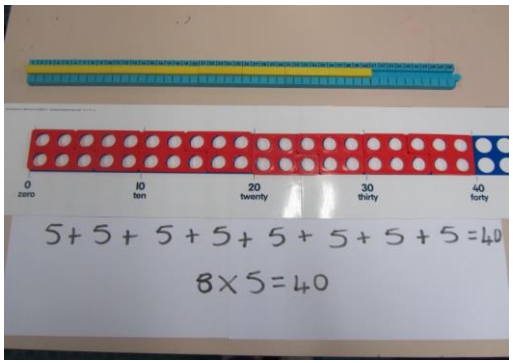
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	 Use practical activities to show how to double a number.	Draw pictures to show how to double a number. 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	 	 Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

	Count in multiples supported by concrete objects in equal groups. 		

Repeated addition

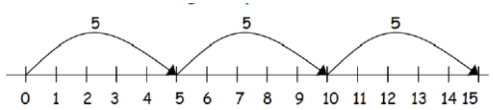


Use different objects to add equal groups.



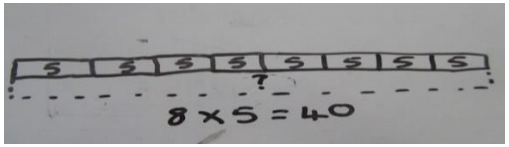
There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?

2 add 2 add 2 equals 6



5 + 5 + 5 = 15

Children can record this as a bar model:



The dotted line shows the unknown quantity. Children could then replace the question mark with the number 40.

Write addition sentences to describe objects and pictures.

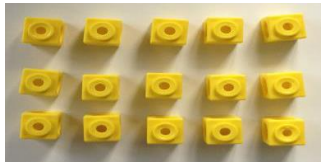
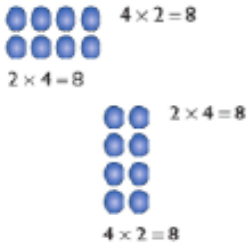
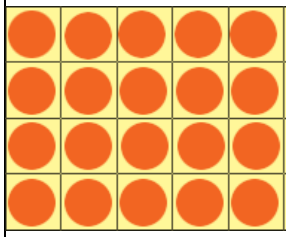
2 + 2 + 2 + 2 + 2 = 10

Children can then record this onto an empty number line:

5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40

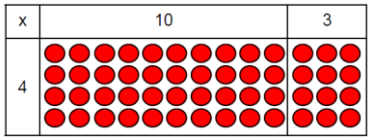
0 5 10 15 20 25 30 35 40

8 x 5 = 40

Arrays - showing commutativity	Create arrays using counters/cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition. Factor x Factor = Product  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 3 + 3 + 3 + 6 = 15 5 x 3 = 15 3 x 5 = 15

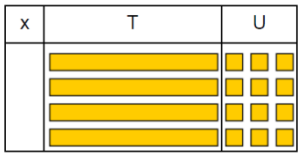
Grid Method

Show the link with arrays to first introduce the grid method.



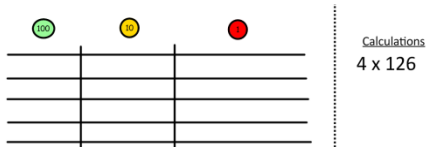
4 rows of 10.
4 rows of 3.

Move on to using Base 10 to move towards a more compact method.



4 rows of 13.

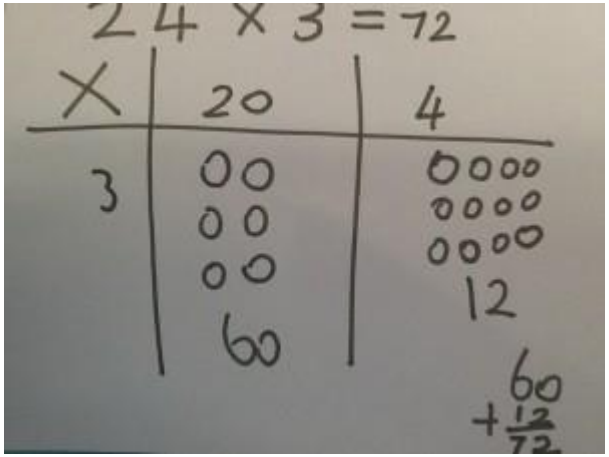
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Fill each row with 126.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

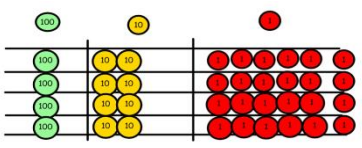
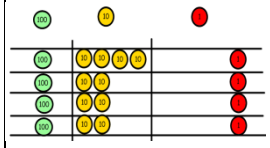
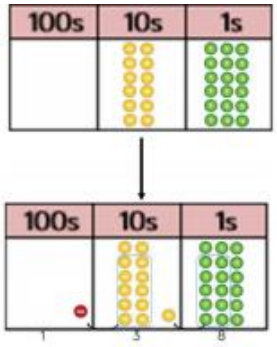
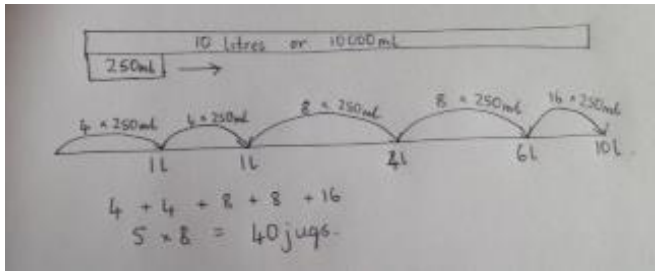
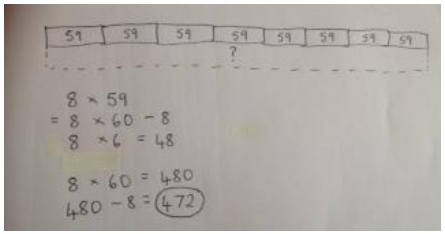
×	30	5
7	210	35

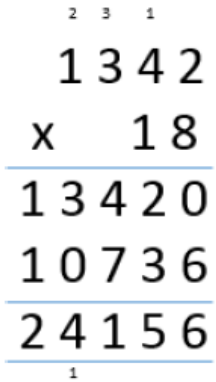
210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

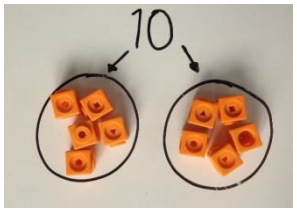
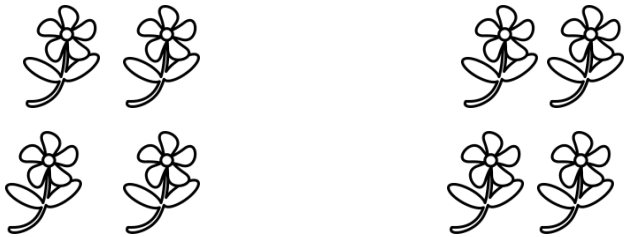
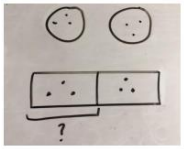
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

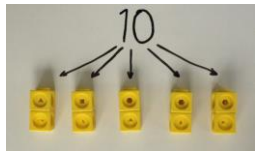
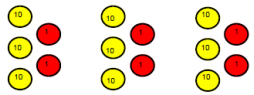
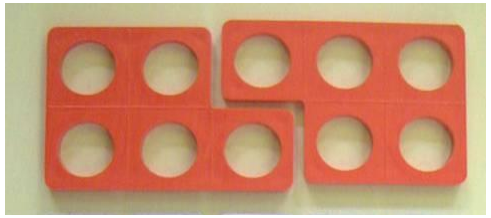
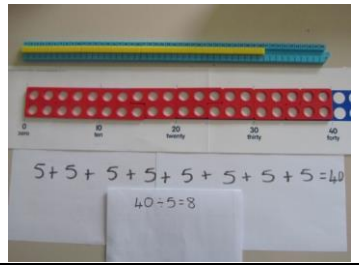
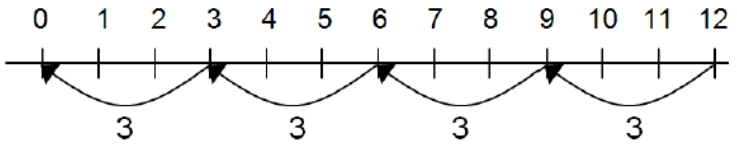
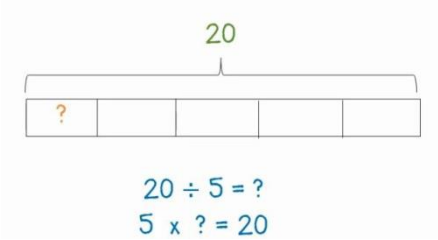
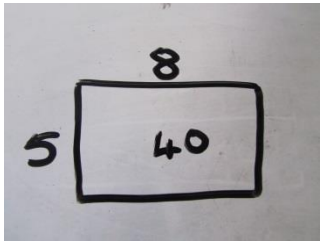
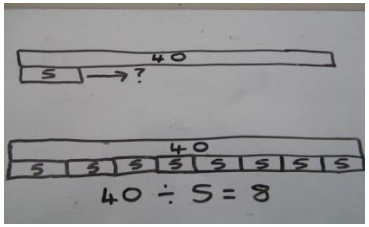
	 Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.   Then you have your product.		
Column Method	Children can continue to be supported by place value counters at this stage of multiplication. 6 x 23 =  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768

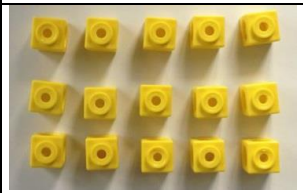
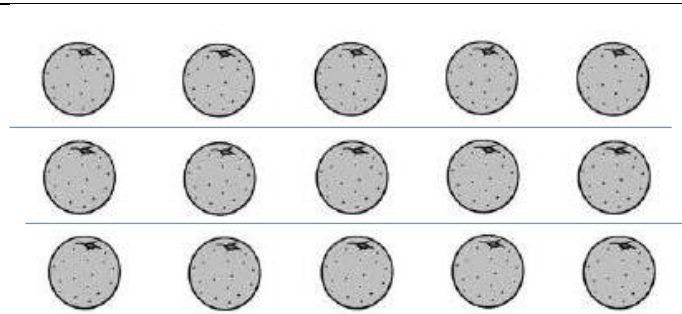
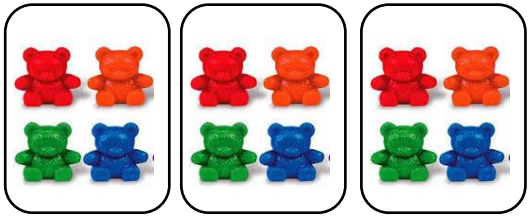
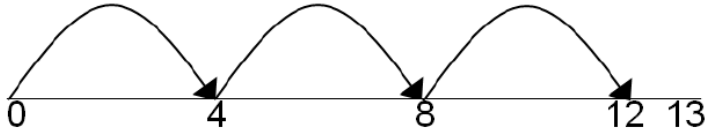
	'With counters, prove that 6 x 23 = 138'	*What is the calculation?* *What is the product?*  	 This moves to the more compact method.  To get 13420 children have solved 1342 x 10. To get 10736 children have solved 1342 x 8.

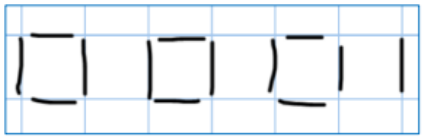
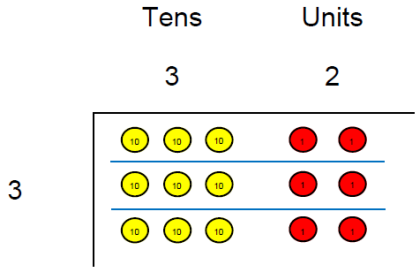
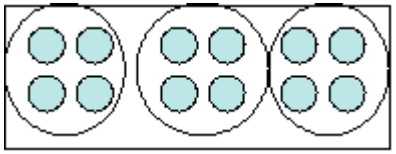
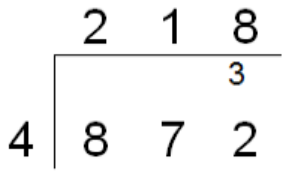
Division

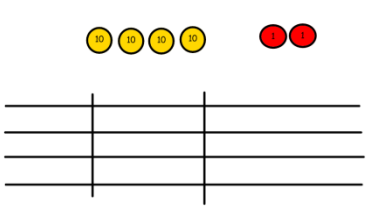
Key Vocabulary: share, group, divide, divided by, half, dividend, divisor, quotient

Objective and Strategies	Concrete	Pictorial	Abstract
Division as sharing.	Sharing using a range of objects.  *I have 10 cubes, can you share them equally in 2 groups?* 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Begin to use mathematical pictures. 	Share 9 buns between three people. 9 ÷ 3 = 3 3 | 3 | 3 Children should be encouraged to use their times tables facts.

Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32    Ten divided into two equal groups.  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.   Children should be taught to represent this as a bar model:  	28 ÷ 7 = 4 *Divide 28 into 7 groups. How many are in each group?*

	How many 5's in 40?		
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. E.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over. 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 	Complete written divisions and show the remainder using r.  13 ÷ 4 = 3 r1

	 Use small sticks/lollipop sticks for 2 digit ÷ 1 digit with remainders. Use lollipop sticks to form wholes. E.g. 13 ÷ 4 squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Draw dots and group them to divide an amount and clearly show a remainder.  Represent lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	Children should be encouraged to use their times tables facts; they could also represent repeated addition on a number line.
Short division	 Use place value counters to divide using the bus stop method alongside an array/grid.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. The use of place value grids with counters drawn in an array should also be used.	Begin with divisions that divide equally with no remainder. 

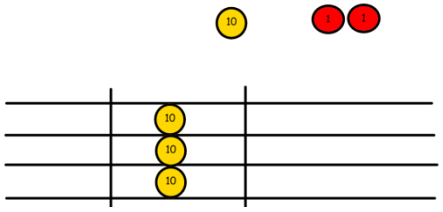


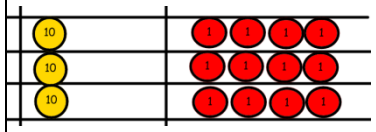
Calculations

42 ÷ 3

42 ÷ 3=

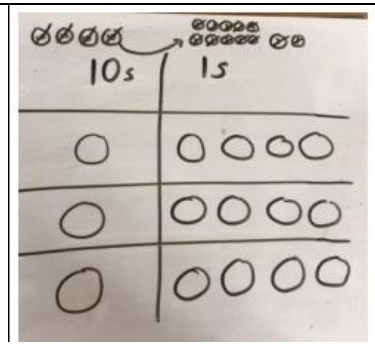
Start with the largest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



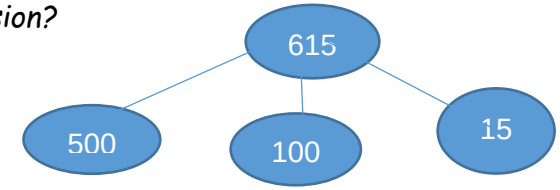


We exchange this ten for ten ones and then share the ones equally among the groups.

We look how much is in 1 group so the answer is 14.



Part whole models can also be used as a variation to the short method. E.g. *Using the part whole model below, how can you divide 615 by 5 without using short division?*



Move onto divisions with a remainder.

$$\begin{array}{r} 8\ 6\ \text{r}\ 2 \\ 5\overline{)\,4\ 3\ 2} \\ \quad\ 3 \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 1\ 4\ .\ 6 \\ 3\ 5\overline{)\,5\ 1\ 1\ .\ 0} \\ \quad\ 16\ \ \ 21 \end{array}$$